LEAPS, METES, AND BOUNDS: INNOVATION LAW AND ITS LOGISTICS

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If you came this way,
Taking any route, starting from anywhere,
At any time or at any season,
It would always be the same . . .

T.S. ELIOT, Little Gidding, in FOUR QUARTETS (1943)†

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INTRODUCTION

This symposium addresses legal policies designed to promote innovation. Economic analysis of technological innovation, diffusion, and decline often proceeds according to sigmoid (S-shaped) models, either directly or as a component in more elaborate mathematical representations of the creative process. The three topics addressed in this symposium—Aereo’s failed attempt to retransmit television broadcasts, agricultural biotechnology, and network neutrality—invite analysis according to one variant or another of the logistic function, the simplest of sigmoid functions. Mindful that mathematics is a philosophical discipline as well as a scientific tool, this Article will extract lessons for the law of innovation from a qualitative application of sigmoid modeling. Innovation and legal policies designed to foster it follow the leaps, metes, and bounds of sigmoid functions.

Part I of this Article introduces the logistic function as the simplest analytical expression of a sigmoid function. Its parameters provide very clear interpretations grounded in physical principles. Part II evaluates the Aereo controversy and agricultural biotechnology as instances of logistic substitution between competing products. The deployment of plant-incorporated protectants and herbicide-resistant crops arguably follows the Hubbert curve, a related function that describes the peak production and eventual exhaustion of depletable resources. Part III proposes

Torrance. Christian Diego Alcocer Argüello provided very capable research assistance. Special thanks to Heather Elaine Worland Chen.


2. See U.S. PATENT & TRADEMARK OFFICE, GENERAL REQUIREMENTS BULLETIN FOR ADMISSION TO THE EXAMINATION FOR REGISTRATION TO PRACTICE IN PATENT CASES BEFORE THE UNITED STATES PATENT AND TRADEMARK OFFICE 7 (2014), available at http://www.uspto.gov/sites/default/files/ip/boards/oed/exam/OED_GRB.pdf (describing mathematics as a philosophical discipline and therefore insufficient by itself to satisfy the technical training requirement for eligibility to take the Patent and Trademark Office examination); see also 37 C.F.R. § 11.7(a)(2)(ii) (2014) (requiring practitioners before the USPTO to “[p]ossess[] the legal, scientific, and technical qualifications necessary . . . to render [patent and trademark] applicants valuable service”); cf. SHARON E. KINGSLAND, MODELING NATURE: EPISODES IN THE HISTORY OF POPULATION ECOLOGY 4-5 (1985) (“On the one hand, knowledge may be sought for purely practical reasons, to predict and control some part of nature for society’s benefit. On the other hand, knowledge may serve more abstract ends for the contemplative soul. Uncovering new relationships is aesthetically satisfying in that it brings order to a chaotic world.”).
multiple ways of understanding network neutrality as a problem of multilayered innovation. The presence of two different types of nonlinear growth, in network operating costs and in expressive diversity, suggests that the law should prescribe independent rather than bundled solutions to these conceptually distinct legal concerns.

I. SIGMOID MODELING AND THE LOGISTIC FUNCTION

A. Sigmoids Across the Sciences

Across the physical, biological, and social sciences, bounded growth processes are modeled according to sigmoid functions. A sigmoid function is a bounded differentiable real function that is supported across the entire domain of real numbers and whose first derivative is either consistently positive or consistently negative.\(^3\) The logistic function and the error function represent two of the most familiar sigmoid functions.\(^4\)

\[
f(t) = \frac{1}{1 + e^{-t}}
\]

\(^3\) See generally Jun Han & Claudio Moraga, The Influence of the Sigmoid Function Parameters on the Speed of Backpropagation Learning, in FROM NATURAL TO ARTIFICIAL NEURAL COMPUTATION 195 (José Mira & Francisco Sandoval eds., 1995).

\(^4\) The following images come from Sigmoid Function, WIKIPEDIA, http://en.wikipedia.org/wiki/Sigmoid_function (last modified Feb. 18, 2015, 5:54 a.m.).
A sigmoid model portrays a system’s potential for accelerated growth at the outset, while simultaneously accounting for negative feedback mechanisms that prevent indefinite, unsustainable growth beyond the system’s carrying capacity. Despite the complexity of many of these processes, many instances of technological growth, substitution, and decline can be elegantly described by a simple mathematical model. The simplest sigmoid model follows the logistic function. The logistic function and the ordinary differential equation for which it supplies an analytical solution play a crucial role in the mathematics of nonlinear, concave growth curves.

Sigmoid modeling in the physical, biological, and social sciences has a celebrated history spanning two centuries. In the tradition of Thomas Malthus’s work on overpopulation and food shortages and Benjamin Gompertz’s demographic “law of human mortality,” nineteenth-century Belgian scientist Pierre-François Verhulst developed a population model based on the intrinsic rate of reproduction \( r \) and an ecosystem’s carrying capacity \( K \).

8. See generally P.-F. Verhulst, Notice sur la Loi que la Population suit dans son Accroissement, 10 CORRESPONDANCE MATHÉMATIQUE ET PHYSIQUE DE L’OBSERVATOIRE DE BRUXELLES 113 (1838) (Fr.); P.-F. Verhulst, Recherches Mathématiques sur la Loi d’Accroissement de la Population, 18 NOUVEAUX MÉMOIRES DE L’ACADÉMIE ROYALE DES SCIENCES ET BELLES-LETTRRES DE BRUXELLES 1 (1845) (Fr.); P.-F. Verhulst, Deuxième Mémoire sur la Loi
Verhulst’s work lay dormant until it was rediscovered in the early twentieth century. T. Brailsford Robertson used the logistic function to model growth in an individual organism and in microbial populations. With evangelistic zeal, Raymond Pearl and Lowell Reed based an entire theory of human populations on the logistic function. With even greater impact, Alfred J. Lotka and Vito Volterra independently extended Verhulst’s model to interspecific competition between predators and prey. The celebrated Lotka–Volterra equations dominated ecology for half a century. In 1950 Theodosius Dobzhansky suggested that climate, especially in the contrast between tropical and temperate regions,
could limit the ecological carrying capacity.\footnote{15} Robert H. MacArthur and E.O. Wilson incorporated the contributions of Verhulst, Lotka, and Volterra into their theory of island biogeography.\footnote{16}

Logistic modeling spread from ecology to the social sciences. From the 1950s onward, economists applied logistic modeling to technological competition and substitution.\footnote{17} By the 1970s, numerous studies on advertising and marketing applied logistic models.\footnote{18} Energy and transportation infrastructure has proved an irresistible subject for logistic analysis.\footnote{19} At sufficiently high levels of abstraction, even sweeping economic transformations such as the Industrial Revolution may be evaluated as the logistic displacement of agricultural labor by mechanical substitutes.\footnote{20}

\footnote{15. See Theodosius Dobzhansky, \textit{Evolution in the Tropics}, 38 AM. SCIENTIST 209, 219-21 (1950).}

\footnote{16. See Robert H. MacArthur & Edward O. Wilson, \textit{The Theory of Island Biogeography} 84, 94-95 (1967).}


B. The Logistic Law of Jurisdynamics

“[T]ime is the longest distance between two places.”21 And nothing as complex and contested as time should be expected to yield its secrets to mathematically crude methods. Economic time series routinely demand empirical tools befitting the full warp and woof of human experience.22 As a form of legal signal processing that is at once sophisticated and tractable, sigmoid modeling offers some hope of “find[ing] in motion what was lost in space.”23

“Anything that begins and ends an existence will fit a logistic.”24 Many phenomena in physics, biology, and the social sciences—from the “population of a species, height of a plant, [or] power of an engine”25 to language acquisition26 and linguistic change27—exhibit an initial spurt of exponential growth. But “natural systems cannot sustain exponential growth indefinitely.”28 Such systems routinely reflect “negative feedback mechanisms or signals from the environment.”29 The interaction between initial exponential growth and negative feedback often generates “a single growth process” following “a single sigmoidal curve.”30

The “three-parameter S-shaped logistic growth model” provides the analytical foundation for projecting technological growth (and decline) according to one or more sigmoid functions.31 S-shaped functions can and should be defined as ordinary differential

23. WILLIAMS, supra note 21, at 115.
27. See PROBABILISTIC LINGUISTICS § 5.3 (Rens Bod, Jennifer Hay & Stefanie Jannedy eds., 2003).
29. Id.
30. Id.
31. Id. at 248 (emphasis omitted).
equations. “Physicists first used [ordinary differential equations] to model the trajectories of moving objects.”32 Ordinary differential equations, “[w]hen applied to populations or technologies,” comparably “describe continuous ‘trajectories’ of growth or decline through time.”33 At their most ambitious, projections of physical and social trajectories give rise to “‘laws of social dynamics’ based on Newton’s laws of mechanics.”34

Consider the following ordinary differential equation:35

\[ \frac{dP(t)}{dt} = \alpha \cdot P(t) \]

where \( P(t) \) represents population as a function of time. The analytical solution to this differential equation reveals a model of exponential growth:

\[ P(t) = \beta e^{\alpha t} \]

where \( \alpha \) is a growth rate constant and \( \beta = P(0) \) is the baseline population at \( t = 0 \).

Although this model might depict the initial stages of seemingly exponential growth, “no bounded system can sustain exponential growth indefinitely unless the parameters or boundaries of the system are changed.”36 Even though a “simple exponential growth model can provide an adequate approximation” for growth during an “initial period,” considerations such as predation and “intraspecific competition for environmental resources such as food and habitat” render “unrealistic” any model that assumes “unrestricted growth.”37 To achieve a more realistic model, we must

32. Id.
33. Id.
34. Id. See generally Elliott W. Montroll, Social Dynamics and the Quantifying of Social Forces, 75 Proc. Nat’l Acad. Sci. 4633 (1978). Expressing linear regression as a first-order ordinary differential equation exposes the limitations of this popular quantitative technique:

\[ \frac{dN}{dt} = \beta \]

The analytical solution to a first-order ordinary differential equation specified as a constant is the family of linear functions following the form, \( N(t) = \beta + C \).
35. Meyer, Yung & Ausubel, supra note 25, at 249.
36. Id.
modify the basic exponential equation “with a limit or a carrying capacity.” The result is a sigmoid curve that resembles the logistic function or the error function.

The simplest and perhaps most widely used sigmoid modification of exponential growth is the logistic function. Analytical expressions of logistic growth are transformations of the basic logistic function:

\[
f(t) = \frac{1}{1 + e^{-t}}
\]

To model logistic rather than exponential growth, we revisit our original formulation of exponential growth as an ordinary differential equation. A logistic growth model adopts the \( P(t) \) and \( \alpha \) terms of the exponential growth function “but adds a ‘negative feedback’ term \( \left( 1 - \frac{P(t)}{\kappa} \right) \) that slows the growth rate of a population as the limit \( \kappa \) is approached”.

\[
\frac{dP(t)}{dt} = \alpha \cdot P(t) \cdot \left( 1 - \frac{P(t)}{\kappa} \right)
\]

The negative feedback term exerts greater limits on the differential equation as \( P(t) \) increases:

\[
1 - \frac{P(t)}{\kappa} \approx 1, \quad P(t) \ll \kappa
\]
\[
1 - \frac{P(t)}{\kappa} \rightarrow 0, \quad P(t) \rightarrow \kappa
\]

To restate the foregoing analysis in English: “[T]he growth rate begins exponentially but then decreases to zero as the population \( P(t) \) approaches the limit \( \kappa \), producing an S-shaped (sigmoidal) growth trajectory.”

The analytical solution to the foregoing differential equation is a logistic function:

38. Meyer, Yung & Ausubel, supra note 25, at 249.
39. See KINGSLAND, supra note 2, at 64-76.
40. Meyer, Yung & Ausubel, supra note 25, at 249.
41. Id.
\[ P(t) = \frac{\kappa}{1 + e^{-\alpha(t - \beta)}} \]

where parameters \( \alpha, \beta, \) and \( \kappa \) are all necessary for the expression of the equation.42 This ordinary differential equation and its analytical solution provide “a parsimonious model [whose] three parameters have clear, physical interpretations.”43 Economic applications of the logistic function are therefore consilient with models applying that function to physical or biological phenomena.44

Growth rate parameter \( \alpha \), which describes the “width or steepness” of the logistic function, is often replaced by the so-called “characteristic duration,” or \( \Delta t \). The characteristic duration \( \Delta t \) specifies the amount of time needed for the function to progress from \( P(t_1) = 0.1\kappa \) to \( P(t_2) = 0.9\kappa \). \( \Delta t \) is a straightforward transformation of \( \alpha \):45

\[ \Delta t = \frac{\ln(81)}{\alpha} \]

The reciprocal relationship between characteristic duration \( \Delta t \) and growth rate \( \alpha \) is precisely what we would expect of time and rate parameters in any mathematical model.

Location parameter \( \beta \) indicates the point in time when the function reaches its midpoint. Formally: \( P(\beta) = \kappa/2 \). Consequently, \( \beta = t_m \), or the midpoint of the logistic function. The standard logistic model “is symmetric around the midpoint \( t_m \).”46 To overcome this limitation, numerous alternative sigmoid models relax the symmetric assumption embedded in the standard logistic function.47

42. Id. at 250.
43. Id. at 248.
45. Meyer, Yung & Ausubel, supra note 25, at 250. Numerically, the natural logarithm of 81 is approximately 4.3944.
46. Id. at 250.
\( \kappa \) indicates “the asymptotic limit that the growth curve approaches,” whether that limit is defined as a “market niche or [as the] carrying capacity” of an ecological system. In a traditional application of the logistic growth model to “the multiplication of bacteria consuming sugar and minerals in a closed petri dish,” \( \kappa \) describes the system’s “carrying capacity,” which “is limited by available space” in the dish. “[S]tagnation” sets in “[a]s the bacteria exhaust the nutritious area of the dish” and “befoul their environment.” The resulting reduction in their rate of growth “produc[es] the S-shaped logistic growth trajectory”:

\[
N(t) = \frac{\kappa}{1 + e^{-\frac{\ln(2)}{\Delta t} (t-t_m)}}
\]

C. Modeling (and Visualizing) Diffusion as Cumulative Adoption

Why growth processes, including the diffusion of innovation, take sigmoid form warrants a brief but dramatic and persuasive

graphic demonstration. In *Diffusion of Innovations*, Everett Rogers recognized that “the rate of adoption for an innovation can be represented by either a bell-shaped (frequency) curve or an S-shaped (cumulative) curve.”“These are just two different ways to display the same data.” Consider the following illustration from *Diffusion of Innovations*, drawn from a study of the uptake of hybrid corn by Iowa farmers:

The sigmoid representation of cumulative adoption held its shape even when a “chi square goodness-of-fit test” showed that “the rate of adoption deviated significantly from a cumulative normal curve . . . in the years 1935 and 1936.”“Nevertheless, the overall rate of adoption over time generally approached a normal S-curve.”

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52. EVERETT M. ROGERS, DIFFUSION OF INNOVATIONS 272 (5th ed. 2003).
53. Id.
54. Id. at 273 (fig. 7-1).
55. See Bryce Ryan & Neal C. Gross, *The Diffusion of Hybrid Seed Corn in Two Iowa Communities*, 8 RURAL SOC. 15, 16-17 (1943).
56. ROGERS, supra note 52, at 274.
as episodic departures from the trend “tend[ed] to cancel one another out over the total diffusion process.”

*Diffusion of Innovations* categorized adopters according to the time at which they took on a new invention.

Using four divisions within the normal Gaussian distribution ($\mu - 2\sigma$, $\mu - \sigma$, $\mu$, and $\mu + \sigma$), Rogers separated adopters into five categories: innovators, early adopters, early majority, late majority, and laggards. As his depiction of the hybrid corn study demonstrated, it is the progression through time of the bell-shaped distribution of adopters that generates the sigmoid cumulative distribution.

Frank Bass developed a more sophisticated variant of Rogers’s model of diffusion. For each product, Bass identified not one but two drivers of adoption: *innovation* attributable to external influence by mass media and *imitation* attributable to interpersonal communications. For our purposes, the practical difference between the models is the greater flexibility of Bass’s approach. Whereas “the Rogers classification . . . assumes the percentage of adopters for the five categories is invariant across innovations, the Bass classification is innovation specific,” and the “percentage of adopters in each . . . categor[y] varies across innovations.”

---

57. Id. at 274-75.
58. Id. at 281 (fig.7-3).
59. See id. at 280-81.
Frequency curves specifying distinct time series by which adopters take up each new innovation generate cumulative sigmoid functions that likewise differ by innovation, with a point of inflection that does not necessarily occur at precisely the fiftieth percentile of the distribution of adopters:63

The logistic distribution used in this Article and favored in much of the biological and economic literature is not materially different from the simpler, purely parametric normal distribution that Everett Rogers used to model the adoption of new technology.64


64. For purely qualitative uses such as this Article, the logistic and Gaussian distributions are essentially equivalent. At a more technical level, these two symmetrical distributions differ in important ways. Whereas the logistic distribution can be specified in closed form with elementary functions, the Gaussian distribution requires use of a special function, the error function. The logistic distribution is slightly more leptokurtic, which makes it a convenient choice (among
Even Frank Bass’s more elaborate model of diffusion relies on parametric statistical boundaries within the temporal distribution adopters. If we adopt the simplest available values for the logistic function’s three parameters—growth rate $\alpha = 1$, midpoint (or location parameter) $\beta = 0$, and carrying capacity $\kappa = 1$—then the simplest variant of the logistic function specifies the cumulative distribution, and its first derivative describes the distribution of adopters:

$$f(t) = \frac{1}{1 + e^{-t}}$$

$$f'(t) = \frac{e^t}{(1 + e^t)^2}$$

Or, in graphic form:

---


66. I generated the following graphic using Wolfram Alpha using the single command line, plot $e^x/(e^x+1)^2$ and $1/(1+e^{-x})$ for $x=-6$ to 6. See WOLFRAMALPHA, http://www.wolframalpha.com/input/?i=plot+%28e^x%2F%28e^x%2B1%29%5E2%20and%201%2F%281%2Be%5E-x%29%20for%20x%3D-6%20to%206 (last visited Apr. 13, 2015).
Logistic analysis enhances our understanding of technological innovation even where, as here, the model is enlisted for purely qualitative use. I have concededly made no effort to gather economic data, let alone to fit them on a formal, fully specified model. But logistic analysis across a wide range of fields has given rise to useful empirical generalizations, or “pattern[s] or regularit[ies] that repeat[] over different circumstances and that can be described simply by mathematical, graphic, or symbolic methods.” As a result, even strictly qualitative interpretation of the logistic model’s physically cogent parameters can provide “rare insights and intuitive understanding” of technological evolution and the law’s proper response to innovation.

II. LOGISTIC MODELS OF TECHNOLOGICAL SUBSTITUTION AND SUCCESSION

A. The Logistic Substitution Model

Perhaps the most common manifestation of logistic analysis in economics and the social sciences, especially in the evaluation of technological diffusion, is the logistic substitution model. This model, pioneered by Fisher and Pry and by Nakicenovic and Marchetti, extends biological work in which Verhulst, Pearl, Lotka, Volterra, and others evaluated organisms and populations according to logistic models. Indeed, the logistic substitution model provides an

67. Cf. Dmitry Kucharavy & Roland De Guio, Application of S-Shaped Curves, 9 PROCEEDIA ENGINEERING 559, 564 (2011) (acknowledging that the majority of uses of sigmoid models involve “pure[ly] qualitative analysis of arbitrary parameters” and that some studies “do not consider any parameters on the vertical scale at all”).

68. Frank M. Bass, The Future of Research in Marketing: Marketing Science, 30 J. MARKETING RES. 1, 2 (1993); see also Mahajan, Muller & Bass, supra note 63, at G79 (observing that the description underlying an empirical generalization “may be approximate rather than exact, and the pattern need not always hold”).

69. Modis, supra note 14, at 869.


elegant way of illustrating the dynamics that govern competition between individual products or even entire lines of technology.72

The logistic substitution model rests upon the following assumptions: First, “[n]ew technologies enter the market and grow at logistic rates.” Second, “[o]nly one technology saturates the market at any given time.” Third, a saturated technology “follows a non-logistic path that connects the period of growth to its subsequent period of decline.” Fourth, “[d]ecaying technologies fade away steadily at logistic rates.”73 Specifically, substitution, once begun, “will proceed to completion,” and the fractional rate at which new technology replaces “old is proportional to the . . . amount of the old” technology that remains.74 “The speed with which a substitution takes place is not a simple measure of the pace of techn[ological] advance . . . .”75 Instead, logistic substitution reflects the imbalances in manufacturing, marketing, and distribution that fuel the eventual displacement of an incumbent technology.76

The first and fourth assumptions suggest that growth and decline can both be modeled in logistic terms, simply by substituting a negative for a positive characteristic duration (the quantity represented by $\Delta t$). The second and third assumptions treat saturation as a phenomenon influenced or even dictated by the emergence and growth of new technologies. The model’s qualitative implications are straightforward: “If a new technology is introduced, its growth must come at the cost (primarily) of the leading technology, causing it to saturate and decline.”77 Mutual rivalry between the technologies is also implicit in the logistic substitution model’s reduction of the standard logistic function’s three parameters to two. In the logistic substitution model, $\kappa$, the carrying capacity, is normalized as 1, or 100% market share.78 Any gains in market share by the new technology necessarily come at the expense of existing technology.79

72. See Meyer, Yung & Ausubel, supra note 25, at 263.
73. Id. at 262; see also Fisher & Pry, supra note 70, at 75.
74. See Fisher & Pry, supra note 70, at 75.
75. Id. at 88.
76. See id.
77. See Meyer, Yung & Ausubel, supra note 25, at 265.
78. See id.
79. See id.
B. Multi-Product Interactions Across Multiple “Broadcasting” Contexts

The logistic substitution model has described and forecast technological interactions as diverse as recorded music media\textsuperscript{80} and natural versus synthetic fibers in clothing.\textsuperscript{81} Meyer, Yung, and Ausubel accurately anticipated that compact disks (CDs) would give way to some other medium for recorded music, but mistakenly predicted (as of 1999) that the replacement technology would be digital versatile disks (DVDs).\textsuperscript{82} Their logistic substitution model proved correct in forecasting the magnitude and timing of CDs’ retreat, but not in identifying the precise technology (MP3 recordings) that would take their place:\textsuperscript{83}

\begin{itemize}
\item \textit{See} id. at 263-66.
\item \textit{See} Fisher & Pry, \textit{supra} note 70, at 77-79.
\item \textit{See} Meyer, Yung & Ausubel, \textit{supra} note 25, at 266.
\item \textit{See} Kucharavy & De Guio, \textit{supra} note 5, at 409-11 (fig.5) (recording media sales with MP3 data to 2003). The data in Kucharavy and De Guio’s figure are plotted semilogarithmically. The so-called Fisher-Pry transform renders logistic data so that it appears linear. \textit{See} Fisher & Pry, \textit{supra} note 70, at 77.
\end{itemize}
In short, even where the logistic substitution model successfully “predict[s] logistic growth and decline, it is a challenging task [to] nam[e] . . . a new technology [in the long-term].”

For its part, Fisher and Pry’s assessment of the clothing fiber market in 1973 appears to have captured only the first among multiple stages of partial logistic substitution of synthetic fibers for cotton. Very crude market data by decade from 1960 through 2010 suggest that cotton has lost market share to synthetic fibers in not one but two consecutive cycles of logistic decline:

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84. Kucharavy & De Guio, supra note 5, at 411.
## World textile fiber consumption, 1960-2010

*Source:* International Cotton Advisory Committee

<table>
<thead>
<tr>
<th>Year</th>
<th>Kilograms per capita</th>
<th>Total consumption in millions of metric tons</th>
<th>Cotton’s share (in %)</th>
<th>Chained logistic model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Cotton</td>
<td>Non-cotton</td>
<td>Cotton</td>
<td>Non-cotton</td>
</tr>
<tr>
<td>1960</td>
<td>3.43</td>
<td>1.59</td>
<td>10.36</td>
<td>4.8</td>
</tr>
<tr>
<td>1970</td>
<td>3.28</td>
<td>2.61</td>
<td>12.11</td>
<td>9.64</td>
</tr>
<tr>
<td>1980</td>
<td>3.23</td>
<td>3.45</td>
<td>14.3</td>
<td>15.29</td>
</tr>
<tr>
<td>1990</td>
<td>3.54</td>
<td>3.67</td>
<td>18.6</td>
<td>19.28</td>
</tr>
<tr>
<td>2000</td>
<td>3.3</td>
<td>4.92</td>
<td>19.98</td>
<td>29.81</td>
</tr>
<tr>
<td>2010</td>
<td>3.29</td>
<td>6.15</td>
<td>22.45</td>
<td>42</td>
</tr>
</tbody>
</table>

\[
\hat{f}(t) = m - \frac{\kappa}{1 + e^{\frac{\ln(81)}{\Delta t} (t - \beta)}}
\]

\[
f(1960) - f(2010) = 33.5\% \text{ (total change in cotton’s market share)}
\]

\[
\frac{f(1960) - f(2010)}{2} = 16.75\% \text{ (half of the total market share change)}
\]

\[
\frac{f(1960) + f(2010)}{2} = 51.55\% \text{ (midpoint of cotton’s market share)}
\]

\[
\kappa = \frac{f(1960) - f(2010)}{2} = 16.75\%
\]

\[
\Delta t = 12.5; \ \alpha = \frac{\ln(81)}{12.5} \approx 0.3516 \text{ (characteristic duration; growth rate)}
\]

for first logistic decline:

\[
m = 51.55\% + \frac{19\kappa}{20} = 67.4625\%
\]

\[
\beta = 1972.5
\]
Leaps, Metes, and Bounds

for second logistic decline:

\[ m = 51.55\% + \frac{K}{20} = 52.3875\% \]

\[ \beta = 1997.5 \]

This chained logistic model is very rudimentary. It draws upon six data points to implement, on a purely parametric basis, two concatenated logistic functions. Its $R$-squared statistic, relative to actual market share data for cotton from 1960 through 2010, is 0.9480.

Notwithstanding the subtleties of logistic forecasting for music and fiber, these markets provide appealing analogies to Aereo’s abortive attack on incumbent multichannel video programming distributors (MVPD)\(^{86}\) and the diffusion of plant-incorporated protectants and herbicide-resistant seeds in agriculture.\(^{87}\) Aereo’s programming platform and Monsanto’s genetically modified seeds represent standard instances of competitive substitution. In each instance, innovators threatened to displace incumbents from saturated markets. In the biological idiom of logistic analysis, Aereo and Monsanto are predators. Their rivals, from incumbent

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broadcasters to suppliers of nonmodified soybean and cotton seeds, are prey.\textsuperscript{88}

The prey–predator relationship between old and new technology represents merely one of four, six, or nine distinct ways in which products can compete with each other.\textsuperscript{89} Logically, interaction between two products that may have positive, zero, or negative effects on either the incumbent product or the entrant can fall into nine categories, as illustrated in the following $3 \times 3$ matrix:

<table>
<thead>
<tr>
<th>Vertical scale: effect of new product on the existing product</th>
<th>Horizontal scale: effect of the existing product on the new product</th>
</tr>
</thead>
<tbody>
<tr>
<td>+ Positive</td>
<td>0 Zero</td>
</tr>
<tr>
<td>+ Positive</td>
<td>Complementary products</td>
</tr>
<tr>
<td>0 Zero</td>
<td>Auxiliary products</td>
</tr>
<tr>
<td>– Negative</td>
<td>Prey–predator product relationship</td>
</tr>
</tbody>
</table>


\textsuperscript{89} \textit{See} Barry L. Bayus, Namwoon Kim & Allan D. Shocker, \textit{Growth Models for Multiproduct Interactions: Current Status and New Directions}, in \textit{New-Product Diffusion Models}, \textit{supra} note 62, at 141, 153-55. The first matrix in the text is derived directly from this source. \textit{See id.} at 155. The second matrix is merely a simplification of the grid provided by Bayus, Kim, and Shocker. On the mathematics of interactions between two products or two biological populations, see generally Felix Albrecht et al., \textit{The Dynamics of Two Interacting Populations}, 46 J. \textit{Mathematical Analysis & Applications} 658 (1974).
We can simplify further by emphasizing only the corners of the table and ignoring multi-product interactions having zero effect on either an incumbent or an entrant:

<table>
<thead>
<tr>
<th>Vertical scale: effect of new product on the existing product</th>
<th>Horizontal scale: effect of the existing product on the new product</th>
</tr>
</thead>
<tbody>
<tr>
<td>+ Positive</td>
<td>– Negative</td>
</tr>
<tr>
<td>+ Positive</td>
<td><em>Complementary products</em></td>
</tr>
<tr>
<td>– Negative</td>
<td><em>Predator–prey relationship</em></td>
</tr>
<tr>
<td>– Negative</td>
<td><em>Prey–predator relationship</em></td>
</tr>
<tr>
<td></td>
<td><em>Product substitutes-in-use</em></td>
</tr>
</tbody>
</table>

A distinct classification of competitive relationships emphasizes combinations (rather than permutations) of “coupling parameters” between two competitors, A and B, based on each competitor’s potential impact on the other’s growth rate:\textsuperscript{90}

<table>
<thead>
<tr>
<th>Mode</th>
<th>Definition</th>
<th>Coupling parameter</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>A</td>
</tr>
<tr>
<td>Pure competition</td>
<td>Each species suffers from the other’s existence.</td>
<td>–</td>
</tr>
<tr>
<td>Predator–prey</td>
<td>One species serves as food for the other.</td>
<td>+</td>
</tr>
<tr>
<td>Mutualism</td>
<td>Symbiosis: A win-win situation.</td>
<td>+</td>
</tr>
<tr>
<td>Commensualism</td>
<td>A parasitic relationship in which one species benefits, but the other remains unaffected.</td>
<td>+</td>
</tr>
<tr>
<td>Amensualism</td>
<td>One species suffers from the existence of the other, which remains impervious to the loss.</td>
<td>–</td>
</tr>
<tr>
<td>Neutralism</td>
<td>No interaction between species.</td>
<td>0</td>
</tr>
</tbody>
</table>

As is evident from any of these classifications of multi-product interactions—either the full $3 \times 3$ or the condensed $2 \times 2$ matrix, or the six combinations of positive, negative, and neutral coupling parameters—the controversies involving Aereo and agricultural biotechnology implicate only the most dramatic of multi-product interactions, the prey–predator relationship in which the introduction of a new product benefits the entrant at the expense of the incumbent. In the industries that straddle both senses of the word broadcasting, which evolved during the twentieth century from a term describing an agricultural technique into a term designating a mass communications medium,91 innovation has simultaneously delivered gains to new technology and ruin to the old.

But no more than the law can bear. “To the economic victor belong only those spoils that may be [lawfully] obtained.”92 Before Aereo, conventional broadcast television had already endured decades of pitched legal battles against cable television.93 Broadcast television has survived the emergence of cable and direct broadcast satellite, due in no small part to the federal government’s repeated efforts, from Southwestern Cable94 through the Turner Broadcasting decisions,95 to shelter conventional broadcasters from unrestrained competition by multichannel distributors.96 A Supreme Court that had given such solicitude to must-carry and retransmission consent regimes unsurprisingly rejected Aereo’s interpretation of the Copyright Act. Despite the federal government’s efforts to preserve the “free” broadcast model against competition by the MVPD market, broadcast television has undoubtedly entered the downward


92. Cf. Rutan v. Republican Party of Ill., 497 U.S. 62, 64 (1990) (“To the victor belong only those spoils that may be constitutionally obtained.”).


sloping phase of the logistic substitution model. The extent of that medium’s decline and its viability in certain niches (such as network news) remain sources of controversy.

C. Peak Glyphosate

The market for broad-spectrum herbicides (and genetically modified crops that resist them) has likewise witnessed multiple cycles of technological rise, decline, and displacement. “Atrazine yesterday, glyphosate today, glufosinate tomorrow.” Agricultural biotechnology does differ from video-programming delivery platforms in a crucial way. Genetic engineering of widely cultivated crop plants eventually reaches a biological limit on the deployment of technology, as target organisms (whether insect pests or weeds) develop resistance under severe selective pressure. More species in an increasing number of locations will evolve their own defenses against any agricultural technology. The proliferation of resistance across biological taxa and geographic space imposes a


ceiling on the usefulness of any plant-incorporated protectant or herbicide-resistance trait that is embedded within crop seeds.

Moreover, technologies differ within agriculture itself. Intervention in plant genetics does not necessarily start a biological countdown to commercial extinction. Hybrid corn, whose uptake during the 1930s inspired one of the most influential lines of research into the innovative process and the diffusion of inventions,\(^\text{102}\) represented the vanguard of an agronomic technique that produced “the predominant form of cultivar in many crops.”\(^\text{103}\) But other forms of agricultural technology have a distinct half-life, as it were. Forms of agricultural biotechnology that exert evolutionary pressure on predators, parasites, or competitors should be regarded as depletable rather than renewable resources. Chief among these technologies are plant-incorporated protectants and herbicide-resistant crop varieties.

To the extent that medical technologies such as antibiotics face similar evolutionary limits on their effectiveness,\(^\text{104}\) apart from economic pressure from rival innovations, those forms of biotechnology should also be evaluated with a similar sensitivity to declines in market share. The mathematical modeling of the diffusion of biological technologies should account not only for the usual problems of logistic substitution, but also for such technologies’ vulnerability to evolutionary pressure.

The leading mathematical model of the rise and fall of depletable resources is the Hubbert curve. Geologist M. King

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\(^{102}\) See Ryan & Gross, supra note 55, at 15; Zvi Griliches, Hybrid Corn: An Exploration in the Economics of Technological Change, 25 Econometrica 501, 501-02 (1957); Zvi Griliches, Research Costs and Social Returns: Hybrid Corn and Related Innovations, 66 J. Pol. Econ. 419, 419 (1958); Zvi Griliches, Hybrid Corn and the Economics of Innovation, 132 Science 275, 275 (1960); Zvi Griliches, Hybrid Corn Revisited: A Reply, 48 Econometrica 1463, 1464 (1980); cf. Zvi Griliches, Research Expenditures, Education, and the Aggregate Agricultural Production Function, 54 Am. Econ. Rev. 961, 961 (1964); see also Rogers, supra note 52, at 54-60 (reviewing the rise and fall of the rural sociology tradition within diffusion literature).


Hubbert predicted in 1956 that peak production of petroleum would signal its eventual exhaustion.\textsuperscript{105} The Hubbert curve is simply the probability distribution function of the logistic distribution. That function is the first derivative of the basic logistic function (which in turn serves as the cumulative distribution function of the logistic distribution).\textsuperscript{106} The formal specification of the Hubbert curve reveals its relationship to the logistic function:

\begin{equation}
    h(t) = \frac{e^t}{(1+e^t)^2} = \frac{d}{(1+e^{-t})} \, dt
\end{equation}

i.e., \( h(t) = f'(t) \), where \( f(t) = \frac{1}{1+e^{-t}} \)


Recognizing the Hubbert peak, or the maximum value of the Hubbert curve, holds the key to predicting when a depletable resource, or a theoretically renewable resource harvested so aggressively as to be depletable, will be exhausted. The Hubbert curve reaches its peak where its first derivative equals zero. One formal specification defines the year of peak production according to the parameters of the Hubbert curve. Cumulative production, \( Q(t) \), may be defined as a logistic function:

\[
Q(t) = \frac{Q_{\text{max}}}{1 + ae^{-bt}}
\]

where \( Q_{\text{max}} \) defines the total available amount of a depletable resource (such as petroleum) and \( a \) and \( b \) are empirically determined constants.\(^{107}\) The year of peak production, \( t_{\text{max}} \), is predicted according to this formula:\(^{108}\)

\[
t_{\text{max}} = \frac{\ln a}{b}
\]

Analysis along these lines predicts that global supplies of phosphorus, a critical ingredient in fertilizer, will peak in 2030 and will be exhausted within 50 to 100 years of the present.\(^ {109}\) Inasmuch as phosphorus is one of three macronutrients in plant fertilizers (along with nitrogen and potassium),\(^ {110}\) the Supreme Court case that anticipates “peak phosphorus” is *Funk Bros. Seed Co. v. Kalo Inoculant Co.*,\(^ {111}\) just as *Bowman v. Monsanto Co.*\(^ {112}\) presages “peak glyphosate.” The Malthusian specter of global famine may yet return

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108. See id.
111. 333 U.S. 127 (1948).
112. 133 S. Ct. 1761 (2013).
in the guise of agricultural asymptotes imposed by absolute limits on exhaustible resources and terrestrial carrying capacity.

It may be possible to model technological successions in agriculture according to either the Hubbert curve or the standard logistic substitution model. The Hubbert curve suggests that peak deployment of a plant-incorporated protectant or an herbicide-resistant crop variety is a function of evolved resistance in target organisms. The logistic substitution model, by contrast, implies that the logistic decline in market share is a function of economic competition between consecutive generations of agricultural technology. In practice, the economic standing of agricultural technologies reflects not only the impact of innovation and adoption, but also any environmental constraints on the effectiveness of those technologies. If logistic growth and decline occur in close succession, then market flow predicted by the logistic substitution model will closely resemble a forecast conducted according to a Hubbert curve.¹¹³

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¹¹³ I generated the following graphic through Wolfram Alpha with the single command line, *plot 1/(1+e^(-x-2)) and 1/(1+e^(x-2)) and 4*e^2/(e^2+1)*e^x/(1+e^x)^2 for x=-6 to 6*. WOLFRAMALPHA, http://www.wolframalpha.com/input/?i=plot+1/(1+e^(-x-2))+and+1/(1+e^(x-2))+and+4*e^2/(e^2+1)*e^x/(1+e^x)^2+for+x=-6+to+6 (last visited Apr. 13, 2015). The blue curve indicates the first half or growth phase of the logistic growth model. The red curve indicates the second half or decline phase of that model. The gold curve indicates a Hubbert curve. The logistic growth model here is indicated by the simplest available parameters: growth rate \( \alpha = \pm 1 \); total market share or carrying capacity \( \tau = 1 \). The location parameter \( \kappa \), which indicates the midpoint of the growth and decline phase, is set to \( \pm 2 \). For its part, the Hubbert curve, which ordinarily reaches its maximum value at \( h(0) = 1/4 \), has been magnified by a factor of \( 4e^2/(e^2+1) \), which is approximately 3.5232, so that the Hubbert peak intersects with both of the logistic curves.
Scaling the Hubbert curve so that it peaks at \( h(0) = 1 \) improves the fit between the two models even more:\footnote{I generated the following graphic with Wolfram Alpha using the single command line, \textit{plot 1/(1+e^(-x-2)) and 1/(1+e^x-2)) and 4*e^x/(1+e^x)^2 for x=-6 to 6}. WOLFRAMALPHA, http://www.wolframalpha.com/input/?i=plot+1%2F%281%2Be%5Ex%29+and+1%2F%281%2Be%5Ex-2%29+and+4*e%5Ex%2F%281%2Be%5Ex%29%5E2+for+x=-6+to+6 (last visited Apr. 13, 2015).}

The exact scaling of these two models, however, is not central to the argument. The point, rather, is that the two models and the
legal narratives they represent are hard to distinguish from one another. The close resemblance between the Hubbert curve and rapid logistic substitution suggests that makers of innovation policy (and, for that matter, environmental regulators) may have trouble discerning whether it is economic competition or evolutionary pressure that has put an agricultural or biomedical technology into eclipse.

D. Inflecting Innovation Policy

Whether technological succession proceeds according to the logistic substitution model or the Hubbert curve, the first step in legally meaningful evaluation of these models consists of determining the point in time on which the technological model pivots. The Hubbert peak marks that moment in the Hubbert curve. In the logistic substitution model, the parameter specified as $\beta$ or $t_m$, either of which designates the inflection point of the logistic function, identifies the key moment. In the standard model of logistic growth, $\beta$ not only indicates the point at which growth reaches half of a system’s carrying capacity—specifically, $P(\beta) = k/2$—but also the moment at which the initial spurt of seemingly exponential growth begins to be overtaken by negative feedback that eventually imposes an asymptotic limit on logistic growth. In more sophisticated extensions of sigmoid modeling, logistic-style growth may not reach its midpoint at the exact center of the time series; that benchmark may be reached either before or after “halftime.” In that event, the real goal is to find inflection point $\beta$, even where $\beta \neq t_m$. The real objective is to ascertain the moment at which initial growth begins to give way to negative feedback and the eventual triumph of resource constraints can be foreseen.

The close mathematical relationship between the logistic substitution model and the Hubbert curve enables us to use the identical mathematical technique to identify the pivotal moment in either model. The Hubbert peak occurs where the first derivative of the Hubbert curve equals zero and the Hubbert curve itself reaches its maximum. Inflection point $\beta$ of a standard logistic growth function is determined according to the second derivative. When the second derivative of a logistic growth function reaches zero, acceleration of the growth rate yields to deceleration. Because the Hubbert curve is the probability density function of a logistic

distribution and the first derivative of a logistic function, the second derivative of a logistic growth function is equivalent to the first derivative of the Hubbert curve:

\[ h(t) = f'(t), \text{ where } f(t) = \frac{1}{1+e^{-t}} \]

\[ \therefore h'(t) = f''(t) = \frac{e^t(e^t - 1)}{(e^t + 1)^3} \]

Or, in graphic terms:\textsuperscript{116}

The plot of \( f''(t) \) makes it clear that \( f''(0) = 0 \) and that \( f''(t) \) is an odd function and is rotationally symmetrical about the origin:

\textsuperscript{116} I plotted the following image through Wolfram Alpha with the single command line, \textit{second derivative of 1/(1+e^(-x)) for -6<x<6}. WOLFRAMALPHA, http://www.wolframalpha.com/input/?i=second+derivative+of+1%2F%281%2Be%5Ex%29+for+-6<x<6 (last visited Apr. 13, 2015). In one step, Wolfram Alpha computed the second derivative of the basic logistic function and plotted it for the range \(-6 < x < 6\).
\[ f^*(-t) = -f^*(t) \]
\[ t < 0, \ f^*(t) > 0 \]
\[ t = 0, \ f^*(t) = 0 \]
\[ f > 0, \ f^*(t) < 0 \]

Knowledge of the location parameter, or inflection point \( \beta \), of a logistic growth curve or of the Hubbert peak immediately enables the computation of the growth rate and characteristic duration of either of these functions. Recall that many specifications of the logistic growth function in economic evaluations of technology replace growth rate \( \alpha \) with characteristic duration \( \Delta t \), where \( \Delta t \) indicates the time needed for a logistic model to grow from one-tenth to nine-tenths of its maximum value, or from 0.1\( \kappa \) to 0.9\( \kappa \). In that instance, \( \Delta t = \ln(81)/\alpha \). Since the logistic substitution model normalizes carrying capacity \( \kappa \) at 1 as an expression of the total market share available to competing technologies, the only parameters necessary to the specification of that model are inflection point \( \beta \) and characteristic duration \( \Delta t \).

In turn, knowledge of characteristic duration \( \Delta t \) provides legally significant insight into the commercial lifespan of a technology. Within this symposium, Yaniv Heled has taken pains to demonstrate that patents are not the only legal tool for conferring economic incentives to innovate.¹¹⁷ For various forms of agricultural biotechnology, regulatory competitive shelters augment or replace patents. However the law elects to incubate innovation, whether by patent or by regulatory competitive shelter, a key question is the appropriate duration.¹¹⁸ If the term of legal protection is too short, prospective inventors may not realize enough of an incentive to develop new technology. If the term of protection is too long, inventors will suppress supplies and gouge consumers, and delays in the transition of patented or otherwise protected technology into the


public domain will retard future innovation.\textsuperscript{119} Dan Burk and Mark Lemley have hinted that patent law should be technology-specific.\textsuperscript{120} Logistic modeling of technological transitions may shed light on questions of timing in the law of innovation, especially the expected economic lifespan of any single invention.

\section*{III. Logistic Analysis Beyond Two-Product Substitution}

The qualitative and quantitative application of logistic analysis to legal subjects is nearly boundless. “In the real world there are many wiggles, speedups, and setbacks, new S-curves growing out of old, separate curves for different sectors and regions of a national economy . . . .”\textsuperscript{121} And even though “[m]ost innovations [do] have an S-shaped rate of adoption . . . the slope of the ‘S’ [varies] from innovation to innovation.”\textsuperscript{122} For those “ideas [that] diffuse relatively rapidly . . . the S-curve is quite steep. Other innovations have a slower rate of adoption, and the S-curve is more gradual, with a slope that is relatively lazy.”\textsuperscript{123} In all instances, the sigmoid function of diffusion “is innovation-specific and system-specific, describing the diffusion of a particular new idea” within a “specific system” or market.\textsuperscript{124} “[O]nly [instances] of successful innovation, in which an

\begin{flushleft}
\textsuperscript{119}. \textit{See} Lim, \textit{supra} note 87 (manuscript at 19-26) (evaluating the legal and economic effects of the expiration of the first wave of Monsanto’s biotechnology patents, especially on its Roundup Ready herbicide-resistant seeds). Of particular concern is the prospect that patentees may try to use licensing restrictions to reclaim rights otherwise extinguished by exhaustion of their patents. \textit{Compare} Fed. Trade Comm’n v. Actavis, Inc., 133 S. Ct. 2223, 2231 (2013) (applying antitrust scrutiny to reverse payments between patent-holding drug developers and their generic competitors even if such payments fell within the scope of the drug patents), \textit{with} Lim, \textit{supra} note 87 (manuscript at 67-72) (outlining the application of \textit{Actavis} to potential anticompetitive behavior by Monsanto after the expiration of patents on the first generation of its Roundup Ready technologies).


\textsuperscript{122}. Rogers, \textit{supra} note 52, at 23.

\textsuperscript{123}. \textit{Id.}

\textsuperscript{124}. \textit{Id.} at 275; \textit{see also} id. at 11 (“[D]iffusion [i]s the process by which (1) an \textit{innovation} (2) is \textit{communicated} through certain \textit{channels} (3) \textit{over time} (4) among the members of a \textit{social system}.”).
\end{flushleft}
innovation spreads to almost all of the potential adopters in a social system,” generate an S-curve.125

The limitations of the simple logistic function (especially its rotational symmetry about the midpoint \( t_m \), where \( P(\beta) = \kappa/2 \)) are easily overcome; the cumulative distribution function of the generalized Weibull distribution can generate a sigmoid model that is either symmetrical or asymmetrical, and one whose inflection point can reflect greater growth in the earlier or later phases of development.126 The fractal nature of logistic functions enables models of this sort to describe exponential growth,127 sinusoid cycling in the tradition of the Lotka–Volterra equations,128 or even stochastic chaos.129 The generalization of the logistic function aspires to treat sigmoid “growth, chaos, self-organization, [and] complex adaptive systems . . . as special cases” of logistic analysis.130

A particularly ambitious variation on this theme, “loglet” analysis, decomposes logistic functions by analogy to wavelet theory as an extension of Fourier analysis.131 (“Loglet” is a portmanteau word that combines “logistic” with “wavelet.”132) Distinct aspects of a single industry, such as natural gas production and consumption in

125. Id. at 275; see also id. (acknowledging that unsuccessful innovations that are “adopted by only a few people . . . [are] ultimately . . . rejected, so that [their] rate[s] of adoption level[] off and, through discontinuance,” plummet).


131. See Meyer, Yung & Ausubel, supra note 25, at 248.

132. Id.
Brazil, lend themselves to exponential, Hubbert, von Bertalanffy, and simple logistic models.  

A. Classifying Network Neutrality as a Problem of Innovation Policy

The expansion of logistic analysis to more ambitious goals demands a broader legal and economic canvas. Product substitution situations such as those presented by Aereo and Monsanto are among the simpler legal issues that sigmoid modeling and regression can inform. The prey–predator relationship in those scenarios represents one of four, six, or nine types of two-product interaction within the economics of innovation. This symposium’s extended discussion of network neutrality provides some hints on the possible expansion of logistic analysis in law beyond basic models of multi-product interaction.

At an absolute minimum, the economic impact of net neutrality may be evaluated according to the assumption that innovation results from the endogenous diffusion of successive generations of a single product. Perhaps broadband infrastructure should be evaluated


134. Cf. Pierre Simon, Marquis de Laplace, A Philosophical Essay on Probabilities 17 (Frederick Wilson Truscott & Frederick Lincoln Emery trans., 1902) (“[T]he more extraordinary the event, the greater the need of its being supported by strong proofs.”).


136. See Bayus, Kim & Shocker, supra note 89, at 144, 146-48 (distinguishing between product diffusion models that emphasize potential interactions among multiple products and models that evaluate the creation and diffusion of successive generations of a single product). If a broadband network is imagined as being successive generations of a single product, each defined by unique features and improvements, then the cooperative approach to innovation acquires greater relevance to the network neutrality debate. See Jorge L. Contreras, Patent Pledges: Between the Public Domain and Market Exclusivity, 2015 MICH. ST. L. REV. 787. Nothing prevents a broadband network operator, after all, from eschewing paid prioritization of traffic, throttling, or any other form of
according to a multi-stage logistic substitution model, much as physical transportation infrastructure in the United States has followed successive generations of logistic diffusion and substitution, from canals to railroads to highways to airports.\footnote{137 See Kucharavy & De Guio, supra note 5, at 413 (fig.7).}

On the other hand, one might argue that any technological progression of this sort has not yet had a significant impact on providers or consumers of broadband service in the United States.\footnote{138 Cf. Nick Russo et al., The Cost of Connectivity 2014, at 12-17 (2014), available at http://static.newamerica.org/attachments/229-the-cost-of-connectivity-2014/OTI_The_Cost_of_Connectivity_2014.pdf (comparing broadband costs in selected cities in the United States, Europe, and Asia).} Cable emerged early as the premier fixed broadband technology in America and has never yielded that dominant position.\footnote{139 See FCC, Connecting America: The National Broadband Plan 42 (2010), available at http://download.broadband.gov/plan/national-broadband-plan-chapter-4-broadband-competition-and-innovation-policy.pdf (predicting that consumers “in areas that include 75% of the population . . . will likely have only one service provider”—namely, cable companies—“that can offer very high peak download speeds”).} From the perspective of residential customers who are effectively locked into a single geographic market, high-speed broadband options are limited; few American consumers have more than a single provider from which to choose.\footnote{140 See David N. Beebe, U.S. Dep’t of Commerce, Competition Among U.S. Broadband Service Providers (2014), available at http://www.esa.doc.gov/sites/default/files/competition-among-us-broadband-
(Mbps) or greater, which the FCC has proposed to set as the definition of high-speed broadband,141 few Americans enjoy a choice among providers.142 Cable broadband has yet to face the sort of competition and resulting erosion of market share that would make it plausible to contemplate logistic substitution of an aging technology whose operator is clinging to a declining position. Eventually, perhaps, municipal broadband networks or even fiber optic networks installed by wealthy private rivals will confront cable operators with the prospect of plummeting profits, stranded investments, and chaotic exit. But that day has not yet arrived.

Charting the progression of high-speed broadband as a story of technological succession would involve extensive data gathering beyond this cursory glance. Moreover, I have spoken so far almost entirely of fixed broadband infrastructure. Rapidly improving mobile devices have expanded the range of tasks that smartphones and tablets may divert from the desktop—and has concomitantly made mobile Internet access a more viable competitor to cable and other fixed broadband platforms. In addition to tracking the rise and ebb of different platforms’ market shares, empirical evaluation of broadband as a possible instance of logistic succession should establish trends in the growth of download and upload speeds, not only in absolute terms, but also in terms of Mbps per dollar spent on monthly subscription fees. In a testy exchange over broadband policy, Christopher Yoo and Susan Crawford embraced one shared technological assumption: cable operators were capable of offering 160 Mbps download speeds in 2006.143 Yet dissenting...
Commissioners could be heard in 2015, decrying the FCC’s proposed threshold of 25 Mbps as aggressive and oppressive.144

B. A Multi-Layered Approach to Logistic Analysis of Network Neutrality

As much as network neutrality has eluded legal classification,145 the neutrality concept also defies easy categorization within innovation policy. It seems naive to force network neutrality, an epochal policy choice decades in the making, into the simple logistic substitution model. At a minimum, cable broadband operators are simultaneously competing on a horizontal basis against potential providers of alternative channels of high-speed Internet access and on a vertical basis against content providers.146 An understanding of the multiple levels of competition at stake begins with an evaluation of the layered nature of broadband networks.

Every communications medium consists of at least three layers: a physical layer consisting of network infrastructure, a logical layer consisting of software and standards for connection, and a content layer.147 Opponents of network neutrality obligations emphasize the capital-intensiveness of network construction, maintenance, and expansion. The specter of “torrent[s] of bandwidth-intensive downstream traffic, such as Internet Protocol Television and other Over the Top applications,” haunts broadband system operators.148 These arguments ring of threats to the physical and logical layers.

By contrast, the competing perspective typically emphasizes end-to-end design in information science, which propels all “intelligence” to the edges of the network (where creators load

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144. See 2015 BROADBAND PROGRESS REPORT, supra note 141, at 111 (Pai, Comm’r, dissenting); id. at 114 (O’Rielly, Comm’r, dissenting).

145. See Verizon v. FCC, 740 F.3d 623, 628 (D.C. Cir. 2014); Comcast Corp. v. FCC, 600 F.3d 642, 645 (D.C. Cir. 2010). During the editing of this symposium, the FCC adopted regulations classifying broadband Internet service as a telecommunications service and imposing network neutrality rules under Title II of the Communications Act. See Protecting and Promoting the Open Internet, 2015 WL 1120110 (2015).

146. See Hurwitz, supra note 135 (manuscript at 4-5).

147. See Yochai Benkler, From Consumers to Users: Shifting the Deeper Structures of Regulation Toward Sustainable Commons and User Access, 52 FED. COMM. L.J. 561, 562 (2000); cf., e.g., Kevin Werbach, A Layered Model for Internet Policy, 1 J. ON TELECOMM. & HIGH TECH. L. 37, 59 (2002) (interjecting a fourth layer, applications, between logic and content).

148. Frieden, supra note 135 (manuscript at 11-12) (footnotes and abbreviations omitted).
content and where consumers access that information) in order to keep the physical and logical architecture of the network as simple and general as possible.\textsuperscript{149} The end-to-end principle’s aspiration of a network with intelligent edges connected by dumb pipe\textsuperscript{150} seeks to “maximize[] innovation” through network “architecture that maximizes the opportunity for innovation.”\textsuperscript{151} In this symposium, Andrew Torrance and Eric von Hippel’s paean to “innovation wetlands” similarly privileges user-initiated innovation over legal regimes that would enable the inventors and guardians of other creative platforms to retain greater control over downstream innovation.\textsuperscript{152}

As a first step toward resolving this debate on the basis of evidence rather than rhetoric, we might conduct logistic analysis of growth rates within the physical and content layers of the Internet. A project that grand exceeds the scope of this Article on a symposium spanning three or four discrete subjects within the law of innovation. According to various formulations of Moore’s law, processing speed in computing doubles every 18, 24, or 36 months.\textsuperscript{153} Moore’s law is therefore a classic instance of exponential growth.

But Moore’s second law, also known as Rock’s law (in honor of investor Arthur Rock), holds that the capital cost of inventing and testing each new generation of semiconductors also rises exponentially.\textsuperscript{154} Logistic analysis posits that there is no such thing as indefinite exponential growth.\textsuperscript{155} As Gordon Moore acknowledged in 2005, on the fortieth anniversary of his original 1965 magazine article describing the periodic doubling of transistor density on integrated circuits, “‘It can’t continue forever. The nature of exponentials is that you push them out and eventually disaster


\textsuperscript{150.} See David S. Isenberg, \textit{The Dawn of the “Stupid Network,”} 2 \textit{Networker} 24, 26 (1998) (describing an end-to-end network as a “stupid network”).


\textsuperscript{155.} See supra text accompanying notes 28-30, 41, and 48-51.
happens.” Serious estimates of the ultimate limits on Moore’s law have ranged from twenty or forty years to 600 years.

Growth within the content layer poses an even more intriguing problem for logistic analysis and cognate forms of nonlinear modeling. Absolute levels of diversity in online content may escape any physical constraint. Perhaps we can find a solution to problems of instantaneous queuing in modern information technology in the deepest of historical studies. Marine paleontology supplies a remote but relevant analogy. Although total biomass on earth is assuredly bounded by the planet’s carrying capacity, biodiversity as measured by the number of distinct species is not necessarily constrained. Logistic models of biodiversity over geologic time assume that limits on ecospace provide negative feedback and impose some ceiling on total levels of diversity. Exponential models assume no such limit and allow the number of species to grow subject only to the ability of biological taxa to occupy new ecospace.

But a third school within paleontology asserts that “the entire Phanerozoic history of marine biodiversity at genus level” is not only unconstrained by putative physical limits on ecospace, but also best described by a hyperbolic growth model whose underlying first-order ordinary differential equation is:


\[ \frac{dN}{dt} = kN^2 \]

Analytical solutions to this differential equation take the general form, \( N(t) = \frac{C}{t_0 - t} \). Unlike logistic growth, which obeys a horizontal asymptote, hyperbolic growth observes a vertical asymptote at the mathematical singularity where \( t = t_0 \).

Paleontology has therefore proposed three mathematically distinct answers to the riddle of diversity. Marine biodiversity at the appropriate taxonomic level (species or genus) may follow logistic, exponential, or hyperbolic growth over the course of geologic history. Although all three of these growth models are convex functions (at least in their initial stages), they behave in dramatically different ways as input grows:

- Logistic growth is constrained: Even as time goes to infinity, logistic growth obeys a finite limit.
- Exponential growth grows to infinity as time goes to infinity, but remains finite as long as time remains finite.
- Hyperbolic growth has a singularity in finite time: It grows to infinity within a finite time.

Or, in graphic form with stylized logistic (blue), exponential (red), and hyperbolic (gold) curves, each scaled so that the function equals 1 at \( t = 0 \):

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166. I plotted the following image using Wolfram Alpha using the single command line, `plot 12/(1+e^(ln(11)-x)) and e^x and 2/(2-x) for x=0 to 1.75`. *WolframAlpha*, http://www.wolframalpha.com/input/?i=plot+12%2F%281%+e%5E%28ln%2811%29-x%29%29+and+e%5Ex+and+2%2F%282-x%29+for+x%3D0+to+1.75 (last visited Apr. 13, 2015).
If expressive diversity online resembles biological diversity in the sense that new forms, artistic or biological, continue to evolve and occupy new niches without regard to physical constraints on ecospace, then online content may be growing at a hyperbolic pace that outstrips even the exponential rates of Moore’s law. Markets, after all, are metaphysical as well as physical spaces, and online expression demands none of the physical variety.

Any decoupling of growth rates in the physical and the content layers of the Internet supports a corresponding decoupling of legal remedies. Whereas growth in content diversity is an unmitigated good, growth in network traffic is not. Congestion and the costs of its management, to say nothing of the capital required to replace the physical infrastructure of existing networks, are the prime movers of incumbent operators’ strident opposition to net neutrality regulation. Usage-based pricing of broadband service can address network operators’ concerns with traffic, stability, and utilization without resort to paid prioritization, the throttling of otherwise lawful content based on its origin, or tiered levels of service—all of which are anathema to net neutrality. But if the costs of network management observe some constraint short of the growth rate of diversity in online content, even if the costs of network operation are rising exponentially within foreseeable time horizons, the case for tying network control to content—as a matter of rhetoric as well as economics—becomes much weaker.

Three counterarguments remain available to opponents of network neutrality. First, and most fundamentally, biological


diversity within ecospace may simply be an inapt basis for modeling, let alone measuring, expressive diversity. Second, as a descriptive matter, it is not at all clear that even biological diversity has grown at hyperbolic rates over the 542 million-year span of the Phanerozoic Eon. In addition to logistic, exponential, and hyperbolic models of species diversity over the course of natural history, a fourth method of evaluating biodiversity argues that the fossil record reveals double periodicity, with marine genera cresting and ebbing according to overlapping 62 million-year and 140 million-year cycles.\(^{169}\) Given the immense number of segments into which the Phanerozoic may be divided, decomposing the entire Eon into discrete components supports “any possible pattern” of exponential, logistic, periodic, or stochastic growth in biodiversity.\(^{170}\) Perhaps it is best just to let the paleontological data speak for itself:\(^{171}\)

\[\text{Biodiversity during the Phanerozoic}\]

\[\text{All Genera} \quad \text{Well-Resolved Genera} \quad \text{Long-Term Trend} \quad \text{The “Big 5” Mass Extinctions} \quad \text{Other Extinction Events}\]

\[\text{Thousands of Genera}\]

\[\text{Millions of Years Ago}\]

\[\text{Phanerozoic, WIKIPEDIA, http://en.wikipedia.org/wiki/Phanerozoic (last modified Mar. 21, 2015, 8:09 a.m.) (drawing from data reported in Rohde & Muller, supra note 169).}\]
Third and finally, wholly apart from the clarity of the paleontological record and its suitability as a guide to the measurement of expressive diversity, there is a colorable argument that network traffic has also grown at a hyperbolic rate. “[W]hen the average arrival rate approaches the server capacity,” “classical queuing theory” predicts “hyperbolic growth” in users’ average waiting time. In the formal notation of queuing theory:

\[ \hat{U} = \frac{\lambda}{\mu}, \text{ where } \lambda = \text{arrival rate and } \mu = \text{service rate} \]

\[ \lambda \to \mu, \quad \hat{U} \to 1 \]

The nightmarish worst-case scenario presented by unchecked growth of Internet traffic is what queuing theory would describe as loss of network stability: impossibly long queues for service, triggering intolerable delays and outright losses of service. Network operation and maintenance under the burden of rules demanding neutrality among different sources of traffic, so the argument proceeds, must labor under a legal prescription “for stealthy low-rate denial-of-service (DoS) attacks inducing arbitrary long queues in . . . target network[s], which in turn cause high delays and loss.”

We might demand more evidence before decrying the “financial ruin” that net neutrality would inflict “upon the simplest


173. See MOSHE ZUKERMAN, INTRODUCTION TO QUEUEING THEORY AND STOCHASTIC TELETRAFFIC MODELS § 3.2 (2015).

174. See, e.g., ROBERT B. COOPER, INTRODUCTION TO QUEUEING THEORY (3d ed. 1990); Robert B. Cooper, Queueing Theory, in ENCYCLOPEDIA OF COMPUTER SCIENCE 1496 (Anthony Ralston, Edwin D. Reilly & David Hemmendinger eds., 4th ed. 2003). Compare Allan Borodin et al., Adversarial Queueing Theory, 48 J. ACM 13, 14 (2001) (“[F]or independent and time-invariant input distributions (say, for example, Poisson arrivals), FIFO [first in, first out] scheduling is stable for any class-independent service time distribution . . . as long as the necessary load conditions (i.e., total expected arrival rate at any server is less than the expected service rate) are satisfied.”), with Maury Bramson, A Stable Queueing Network with Unstable Fluid Model, 9 ANNALS APPLIED PROBABILITY 818, 818 (1999) (identifying “a family of queueing networks that are stable, but whose fluid models are unstable, that is, there exists an unstable solution of the fluid model equations”).

175. See BERGER, KARSTEN & SCHMITT, supra note 172, at 1.
Internet service provider] who finds his [network] conscripted to national [informational] use.”176 The burden of persuasion, in my judgment, remains with opponents of net neutrality. Despite this debate’s superficial emphasis on network management and incentives to maintain, expand, and build digital networks, the real innovative stakes reside in the content layer. The ultimate question is whether independent suppliers or the networks themselves will excel in meeting demand for novel, engaging content. In his contribution to this symposium, Thomas Jeitschko emphasizes economic distinctions between patents and other forms of property and urges makers of innovation policy not to equate intellectual property with rights in land or other tangible property.177 In a symposium dominated by considerations of economic rivalry and resource-based constraints, diversity in the Internet’s content layer stands out as the lone element of innovation that potentially heeds no carrying capacity.

IV. LAW’S ECOLOGY

Qualitative evaluation of the debate over network neutrality, as informed by the quantitative insights of logistic analysis, leaves this controversy in the deep ideological trenches dug by partisans in American debates over innovation policy. Network operators preach Joseph Schumpeter’s gospel of creative destruction through monopoly, or at least through the quest for supracompetitive returns.178 Creators of content enjoying neither control of the network


nor an affiliation with any network operator invoke Kenneth Arrow’s competing vision of innovation through robust competition.\(^{179}\)

The unrelenting battle between Schumpeterian and Arrovian accounts of innovation,\(^ {180}\) so pivotal to debates over the optimal scope of intellectual property rights,\(^ {181}\) reprises debates over \(r/K\) selection theory in ecology and evolutionary biology. \(r\)-selection in biological species (analogous to Arrovian competition) favors low-cost reproduction of numerous offspring, while \(K\)-selection (analogous to Schumpeterian competition) favors the expenditure of enormous energy in the production of a low number of high-quality offspring.\(^ {182}\) The \(r\)- and \(K\)-selection strategies derive their names from the ecological literature’s preferred rendering of the differential form of the logistic function:

\[
\frac{dN}{dt} = rN \left(1 - \frac{N}{K}\right)
\]

\(r\) and \(K\) are the preferred names in ecology and evolutionary biology for the growth rate and carrying capacity parameters we have designated as \(a\) and \(\kappa\).\(^ {183}\)

\[\text{"[T]he end of all our exploring / Will be to arrive where we started / And know the place for the first time."}^{184}\]

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183. See, e.g., Rosenzweig & MacArthur, supra note 14, at 217. Compare *Kingsland*, supra note 2, at 74-75 (extolling “the differential form” of “the logistic curve” as “easier to interpret and to analyze” than its differentiated, analytical version), with *id.* at 85-86 (describing Alfred Lotka’s rendering of the differential form of the logistic function according to \(r\), the function’s rate of increase).
This Article’s abbreviated and strictly qualitative application of sigmoid models presents merely a few special cases of generalized logistic functions. It represents a modest contribution to the “larger effort to move legal science toward a first law of jurisdynamics.” Law, like ecology, is a “search for patterns of repetition.” The enterprise is fraught with some danger: Just as the invocation of the word law in biology “sets off emotional resonances,” as though the announcement of a scientific law “suggest[s] that we have discovered a truth of some importance,” the reverse process of intellectual osmosis—the infusion of concepts from the physical and biological sciences into law—sets its own philosophical trap. Whenever “ideas taken from physics have been transferred to a biological context,” including the life science called law, proponents of these ideas risk “reject[ing] . . . history in favor of” populations and markets “in equilibrium” as “a harmonious, unifying concept.”

We can save through motion what we might otherwise lose to stasis:

The detail of the pattern is movement, . . .
Desire itself is movement
Not in itself desirable;
Love is itself unmoving,
Only the cause and end of movement,
Timeless, and undesiring
Except in the aspect of time
Caught in the form of limitation
Between un-being and being.

The consilient application of logistic analysis to cycles of growth, competition, decay, and renewal supports a general system theory of physics, biology, and human society. To better


184. ELIOT, supra note 1, at 59.
185. See Banks, supra note 47; Tsoularis, supra note 37.
188. KINGSLAND, supra note 2, at 68.
189. Id. at 8.
190. ELIOT, Burnt Norton, in FOUR QUARTETS, supra note 1, at 13, 19-20.
191. See generally LUDWIG VON BERTALANFFY, GENERAL SYSTEM THEORY: FOUNDATIONS, DEVELOPMENT, APPLICATIONS (1968).
“organiz[e] our understanding of economic, ecological, and institutional systems,” such a theory would exploit “notions of hierarchies across scales to represent structures that sustain experiments, test results, and allow adaptive evolution.” At its most ambitious, logistic analysis in law extends $r/K$ selection theory beyond its venerable but increasingly brittle origins in biology into a more comprehensive life-history paradigm or even an all-encompassing theory of “panarchy,” on earth as in the heavens.


195. *See* *PANARCHY*, *supra* note 192.
